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(Review Article)

Geometry: An innate ability that nevertheless makes it difficult for students worldwide

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Abstract

The importance of geometry has been highlighted since antiquity. Studies show that we have geometric knowledge from infancy age. Some geometric skills are innate and are not taught in school. However, cultivating them can be achieved in many ways at a very young age and has positive academic results in the future. Unfortunately, nowadays students do not show the necessary interest in the subject and as a result they have not developed their geometric thinking and encounter difficulties both in everyday life and at school.

Keywords: Innate geometry; Construction toys; Geometry disabilities; Students

1. Introduction

The roots of geometry go back many centuries. According to historians, the Egyptians first discovered geometry in an attempt to measure the area of their fields, which the Nile flooded every year. That is where Geometry, "Earth" and "measurement", got its name. In addition to the ancient Egyptians, the Babylonians, the Chinese, and the Indians studied and applied Geometry. All of these people may have applied Geometry, but they were empirical they did not follow any rules and did not try to give logical proofs, the goal was to solve problems. The revolution was brought about by the Greek mathematicians, who tried to fill these gaps. It was Thales of Miletus who passed on this knowledge to Greece after his journey to Egypt and Mesopotamia. During his stay in Egypt, he managed to measure the actual height of the pyramid of Giza, with the help of geometry, which was an unsolvable problem even for its builders. Many more Greeks followed, such as Mamercus (or Mamertius), Pythagoras, Plato, Euclid, and Archimedes. But the father of Geometry can be considered Euclid, who compiled all the Elements.

Experiments in infants, rats, monkeys, and even fish and insects provide evidence that basic knowledge of geometry is innate (Cheng & Newcombe, 2005; De Cruz, 2009a; Vallortigara et al., 2009). There is research claiming that our cognitive system pays particular attention to the spatial invariants it extracts from the environment; this is proto-geometric knowledge. Featural properties of our environment change significantly over time, unlike geometric properties, such as the distance between two trees (Ferreirós & García-Pérez, 2020a).

Another proof that some geometric concepts are innate in man is provided by Socrates, a Greek-born scientist. He poses the following problem to a slave, Menon. He presents Menon with a square of side 2 feet and area 4 and asks him how much the side of a square of area 8 feet will be. The slave gives the correct answer without having been taught geometry.

Dehaene and his colleagues (2006) conducted a study that confirms that some geometric concepts are innate in humans. The study involved participants from an Amazonian tribe and North Americans. Participants were given two non-verbal tests that included geometric concepts (e.g. parallelism, squares, triangles, symmetry, etc.) without using the terms, but

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rather examples from everyday life. One of the tests given presented six pictures of a geometric concept (e.g. parallelism) of which five pictures were correct and the sixth picture was wrong, and the aim was to identify the picture that did not match the others (Figure 1). Amazonian individuals scored exceptionally well on topology concepts (e.g., connectedness)), Euclidean geometry (e.g., lines, points, parallelism, and right angles), and geometric shapes (e.g., squares, triangles, and circles). The researchers concluded that uneducated adults and children from an isolated culture have similar abilities in basic geometric concepts as children from Western culture. The researchers concluded that knowledge possessed by all groups except educated American adults is common in core geometry (Dehaene et al., 2006)The results of this research are in line with other studies, and validate that some aspects of geometry are "innate" and do not depend on culture or teaching (Chiandetti & Vallortigara, 2008; De Cruz, 2009b; Ferreirós & García-Pérez, 2020b; Pica et al., 2004; Rosch, 1975; E. Spelke et al., 2010; E. S. Spelke, 2011).

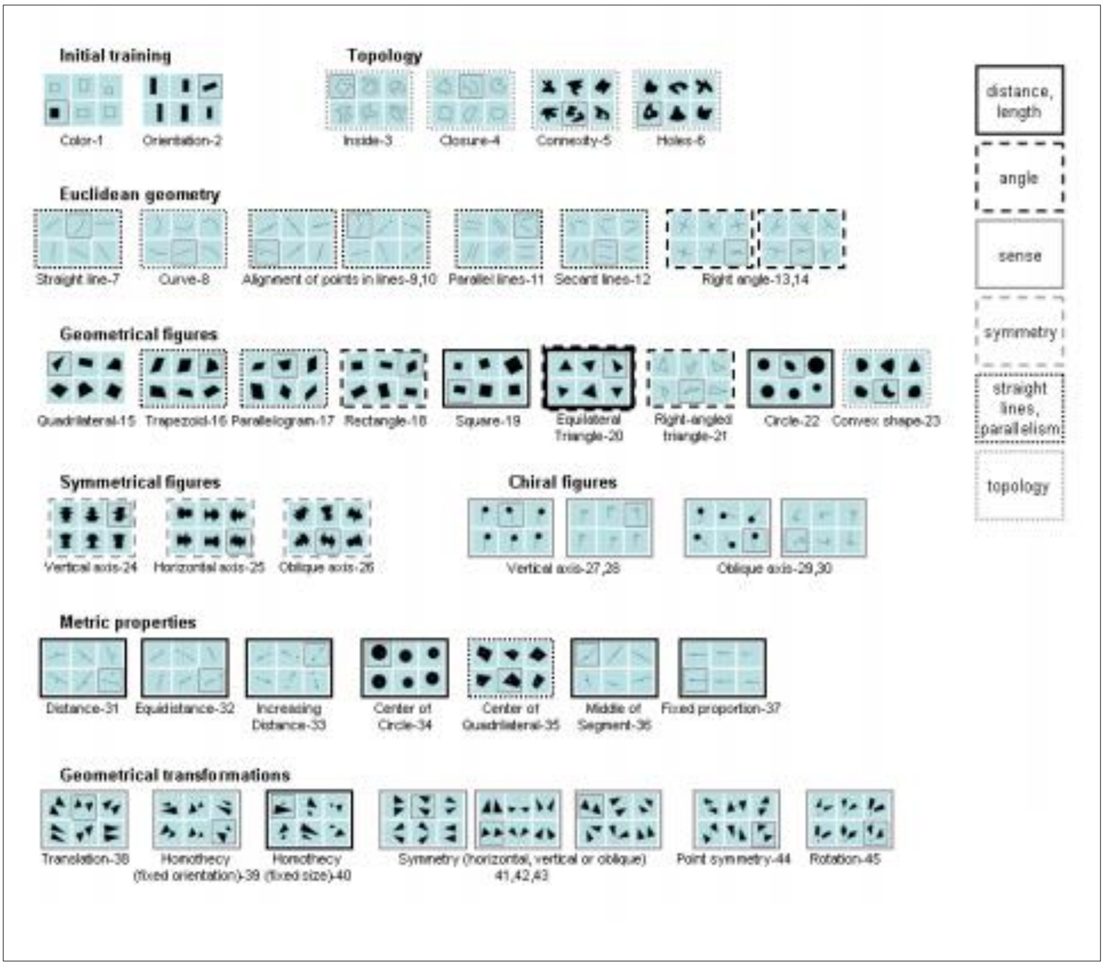

Figure 1 The non-verbal tests to which the students were subjected

We can conclude that certain aspects of geometry are:

- primary (Rosch, 1975),
- have been developed from a very early age (Lourenco & Huttenlocher, 2008a; E. Spelke et al., 2010); and
- not dependent on culture and teaching (Dehaene et al., 2006).

2. Understanding geometric concepts from infancy to adulthood

In the 1990s, a more systematic attempt was made to study infants' sensitivity to geometric relationships. These studies revealed that newborn infants can distinguish differences between line segments of different lengths and between angles of different apertures(Dillon et al., 2020; Slater et al., 1990, 1991; Tsuruhara et al., 2009). Sensitivity to length and angle is strong throughout infancy and is observed even when objects change orientation or position. In contrast to their ability to process length and angle, infants do not appear to recognize the directional property in a small-scale two-dimensional figure (Izard et al., 2009; Lourenco & Huttenlocher, 2008b). At age six children can extract information about angles with an accuracy comparable to their understanding of distance and directional relations (Ferreirós & García-Pérez, 2020b).

According to the Common Core State Standards (CCSS,2010), students should begin to gain experience in geometry in kindergarten and move on to use more precise definitions and develop careful evidence throughout their primary and secondary mathematics education. For example, toddlers are expected to identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres) and by third grade, students should be able to reason with the help of shapes and their characteristics (Zhang et al., 2012).

Biederman and Cooper (1991) showed that the processing of a shape is facilitated if it is preceded by a presentation of the same shape or its symmetric, with a similar rate of facilitation in both cases. These findings suggest that adults possess a basic shape recognition process, based on angle and length, that is impervious to discrimination of form. However, they learn to overcome a limit to this process and identify the relationships of form to alphanumeric characters when they learn to read (e.g., e to 3) (Biederman & Cooper, 1991).

Researchers have found that geometry is learned through associative or rapid adaptive learning processes, applied to the spatially structured world projected to our senses. Consistent with these projections, animals learn to use arbitrary landmarks as guides to environmental locations (Cheng & Newcombe, 2005), children's spatially guided behavior shows regular increases in accuracy with growth and experience (Spencer & Hund, 2003), and adult navigation is systematically altered by experience (E. Spelke et al., 2010).

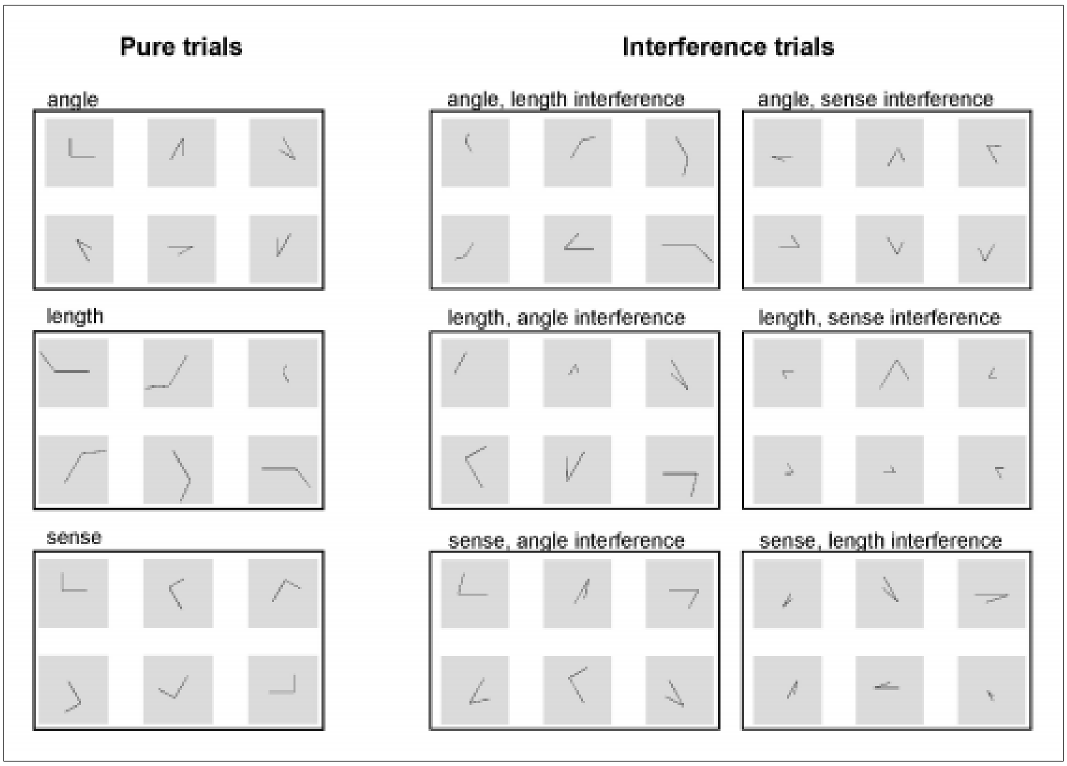

Figure 2 Activities submitted during the third experiment(Izard & Spelke, 2009)

In addition, activities such as Lego building have been found to be related to mathematical performance (Ahmadi, 2021; Hussain et al., 2006; McDougal et al., 2023a, 2023b; Murti & Szucs, n.d.; Nath & Szücs, 2014a, 2014b; Rejeki et al., 1848; Taylor, 1989; C. Wolfgang et al., 2003) as have the games that children play at an early age. The association between construction games and mathematics performance in children was not significant at an early age, but this association appeared to become significant at age 12 and above. This demonstrates that construction play in kindergarten may have long-term effects on logical-mathematical knowledge (C. H. Wolfgang et al., 2001).

Izard and Spelke, in 2009, conducted three experiments that evaluated the development of sensitivity to different types of geometric information in visual shapes, with a particular focus on length, angle, and time. The findings showed that children up to 4 years of age analyzed shapes by detecting distance and angle relationships, but were unable to detect the relationships that distinguish an object from its symmetric. Patterns of visual shape analysis showed high developmental invariance: properties that were less detectable by children. In general, sensitivity to all tested properties improved with age. While, for all groups aged 12 to 17 years, performance did not differ from adults. When presented with a carefully controlled set of shapes varying exclusively in length, angle or direction, children were found to develop sensitivity to these properties at different rates, responding first to length, then to angle and last to direction. Between the ages of 8 and 10, children develop a specific sensitivity to right angles, which they begin to regard as distinct. Geometric ability thus appears to occur as an interaction between developmentally invariant, basic intuitions and later acquired distinctions (Izard & Spelke, 2009).

Recent evidence suggests the distinction between intuitive geometry and academic geometry. Intuitive geometric concepts (e.g. Euclidean geometry) are shared by people regardless of culture and formal education (Dehaene et al., 2006; E. Spelke et al., 2010). In contrast, academic geometry, operationally defined as the ability to answer formal geometric questions and problems encountered in schools (Giofrè et al., 2014)involves concepts that are primarily learned through formal instruction. Academic geometry requires explicit knowledge of principles and concepts (e.g., diagonals, parallel lines, and right angles) and their rules and applications in representing complex spatial relationships (e.g., imagining the result of combining two shapes). Such learning also involves applying rules to specific requests (e.g., calculating the area or perimeter of a shape) (Giofré et al., 2014).

3. Van Hiele learning model

Pierre Van Hiele and Din Van Hiele-Geldof attempted to construct a theory that could level students' geometric perception and help them develop skills such as visualization, critical thinking, intuition, perspective, problem-solving, conjecture, deductive reasoning, logic, argumentation and proof. The van Hiele model of geometric thinking emerged from the doctoral works of two Dutch mathematicians, Dina van Hiele-Geldof (1984) and Pierre van Hiele (1984), in addition to the levels they focused on the phases of instruction.

The van Hiele theory originally consisted of five successive and hierarchically distinct levels of geometric thinking (Usiskin, 1982). In modern literature, however, one can find two different ways of numbering van Hiele levels: levels 0 to 4 and levels 1 to 5 (Galitskaya & Drigas, 2020). The original van Hieles numbering version used Levels 0 to 4, however, American researchers Mason (1998), Usiskin, (1982) and van Hiele (1986, 1999) replaced the numbering with 1 to 5. This according to Mason (1998) allows for a sixth level, pre-recognition (pre-visualization) level,i.e. level for learners who have not even achieved the basic Level 1 and they called it Level 0 (Bashiru & Nyarko, 2019; Burger & Shaughnessy, 1986; Halat, 2008; Halat & Sahin, 2008; Mason, 2009).

Van Hiele pointed out that the transition from one level to another requires a period of time longer than an hour or a teaching unit. Dina (1957) in a study of 12-year-old students reported that to move from level 1 to level 2 took 20 lessons and from level 2 to 3, 50 lessons (Usiskin, 1982).

Many researchers based on the Van Hiele model have attempted to test students' level in geometry. Unfortunately, the results showed that students do not possess the skills they should have and this is true globally.

In 2019, a similar survey of 105 students was conducted in schools in Ghana by Bashiru and Nyarko. Four secondary schools participated in the survey of which two were private. Most students (65) possessed skills corresponding to level 1, only 17 students possessed level 2 skills and only one student reached level 3. The remaining students could not even respond to activities corresponding to level 1, so they were at level 0 (Bashiru & Nyarko, 2019).

These findings are also in agreement with other studies conducted (Mensah et al., 2023; Prayito et al., 2019; Rachmaniah Mirza Hariastuti, 2023; Škrbec & Čadež, 2015).

Jiří Haviger and Iva Vojkůvková conducted a study involving secondary school students aged 15-17 years, which aimed to test the level of van Hiele's geometric thinking of Czech students. They found that Czech geometric education emphasizes the teaching of levels 1 and 2, and partly level 3. Their findings were as follows (Haviger & Vojkůvková, 2015)

- 97% of students achieved level 1, this level for the age group (15-17 years old) is easy.
- 87% of the students achieved level 2, this level is slightly more difficult than level 1.
- To achieve level 2 requires knowledge of level 1 (the percentage of students who achieved level 2 without level 1 was 2.6%).

- Level 2 logically follows on from Level 1.
- 39% of students achieved level 3. This level is most helpful in separating students of age (15-17 years) into classes.
- Level 2 presupposes level 2 (only 8.7% skip level 2).
- Level 3 logically follows level 2.
- 9% of students managed to achieve level 4. The level of difficulty is higher than level 3.
- 47.4% of students managed to achieve level 4 by skipping level 3. So they concluded that level 4 does not logically follow level 3.
- The understanding and use of geometric shapes is important to be mastered by all students as they are used in other areas of mathematics (fractions, multiplication in arithmetic, 2D and 3D graphs and functions, and statistics) (Mathematics, 2009).

4. Difficulties and geometry

Students attending secondary schools need to master concepts, definitions, theorems, etc. and apply their knowledge to solve problems usually presented in a linguistic form (Giofrè et al., 2013). Based on Ozerem's (2012) study, 7th-grade students have difficulties in geometry and these difficulties are due to students' weak memorization towards geometric formulas or concepts, and students' inability to relate geometry to real life. Other causes of students' difficulties in learning geometry were the weak knowledge of the concept, weak understanding of mathematical skills and students' lack of enthusiasm towards the discipline (Gloria, 2015).

H visual-spatial and verbal-logical thought correlate with success in geometry and problem-solving.(Battista, 1990). Saad and Davis (1997) reported that language skills affect performance in geometry (Saads & Davis, 1997). Kikovich (2015) found that the difficulties students experience in geometry are related to their understanding of mathematical language and its connection to preliminary knowledge (Kivkovich, 2015). Lean & Clements (1981) argued that to achieve better performance in geometry and 3D representations, one must have spatial skills and knowledge of spatial conventions (Lean & Clements, 1981). Battista (1990) argued that students who possess a high level of geometric thinking more often use an analytical rather than a visual approach for problem-solving (Battista, 1990).

Solving mathematical word problems is a complex process that involves multiple cognitive processes (Pearce et al., 2013; Pongsakdi et al., 2020; Swanson et al., 2015a). To solve word problems, students must first understand the information presented within the problem, create a mental model of the problem with the information provided, and then determine a pathway for solving that is based on their chosen model (Gonsalves & Krawec, 2014; Montague, 2003). In addition, the role of working memory is important, as each of the steps to the solving require students to access pre-stored information, select the appropriate algorithm, and apply problem-solving procedures to a variety of real-world situations (Swanson, 2015, 2016; Swanson et al., 2015b).

Some researchers have argued that students' particular difficulties in geometry may represent a unique and specific secondary type of learning difficulty(Chen et al., 2021), while others that performance in geometry is not independent of performance in other types of mathematics (Peng et al., 2018).

However, it is important to note that research shows that academic achievement in geometry (secondary school), is considered to be linked to a student's future academic and professional success (Verstijnen et al., 1998).

5. The role of ICTs - Discussion

Students have grown up with technology and the use of ICT is now essential in everyday life. The use of specific software in the teaching of geometry has proven to be very effective and is recommended by many researchers (Adelabu et al., 2019; Bhagat & Chang, 2015; Galitskaya & Drigas, 2019; Kim & Md-Ali, 2017; Lu'luilmaknun et al., 2021; Nzaramyimana et al., 2021; Rashevska et al., 2020; Sumarwati et al., 2020; Zaranis, 2014). Numerous studies present new mobile applications (A. Drigas et al., 2020; Stathopoulou et al., 2019)and intervention programs that can improve student performance in both special and general education (Bindu, 2016; Carnoy, 2004; Das, 2019; Doulou & Drigas, 2022; A. Drigas & Charami, 2014; A. Drigas & Kokkalia, 2014; A. Drigas & Kostas, 2014; A. Drigas & Petrova, 2014; A. Drigas & Rodi, 2013a, A. S. Drigas & Ioannidou, 2013; A. Drigas & Vlachou, 2016; Flecknoe, 2002; Henderson, 2020; Majumdar, 2015; Noor-Ul-Amin, 2013; Xanthopoulou et al., 2019; Zweekhorst & Maas, 2015). These programs often incorporate artificial intelligence (AI), virtual reality (VR), STEM, or robotics(Anagnostopoulou et al., 2020; Brainin et al., 2021; Buckley et al., 2018; L. Chen et al., 2020; X. Chen et al., 2020, 2022; Demertzi et al., 2018; Di Lieto et al., 2020; Dorouka et al., 2020; A. Drigas et al., 2022b; A. Drigas & Dourou, 2013; Hopcan et al., 2023; Huang et al., 2021; Hutchins et al., n.d.; Julià & Antolì, 2018; Kefalis & Drigas, 2019; Luan et al., 2020; Lytra & Drigas, 2021; Mitsea et al., 2020; Ng et al., 2021; Ouyang & Jiao, 2021; Pappas et al., 2018, 2019; Pratama et al., 2023; Sisman et al., 2021; Su & Yang, 2022; Vincent-Lancrin & Van der Vlies, 2020; Xie, 2020; Yang, 2022; Zhai et al., 2021; Zhong & Xia, 2020). All of these techniques and practices improves the emotional educational procedures and provide students with a friendly and enjoyable environment (Chaidi & Drigas, 2022b). Many researchers suggest that for better and faster results, ICT can be combined with theories and models of metacognition, mindfulness, meditation and the cultivation of emotional intelligence (Bamicha & Drigas, 2022; Chaidi & Drigas, 2022b, 2022a; A. Drigas et al., 2017, 2022a, 2022c; A. Drigas, Mitsea, et al., 2021; A. Drigas, Papoutsi, et al., 2021; A. Drigas & Papoutsi, 2021; A. Drigas & Sideraki, 2021; Galitskaya & Drigas, 2021; Mitsea et al., 2022)

6. Conclusion

Based on the literature review we found studies that prove that geometric' s skills is innate in humans other studies prove that babies have geometric skills such as distinguishing differences between straight segments and angles. These skills can be developed with the help of construction toys such as Lego. Unfortunately, there is a disregard for geometry as students have not been successful in developing the skills required and this phenomenon is occurring worldwide. Students claim an inability to learn geometric formulas and concepts, an inability to relate geometry to the surrounding space, a lack of interest in the subject as well as an inability to understand the problem and retrieve the knowledge necessary to solve it.

Compliance with ethical standards

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Disclosure of conflict of interest

The Authors proclaim no conflict of interest.

References

[1] Altieri, M., & Nicholls, C. (2018). Biodiversity and pest management in agroecosystems. Boca Raton: CRC Press.

[2] Adelabu, F. M., Makgato, M., & Ramaligela, M. S. (2019). The importance of dynamic geometry computer software on learners' performance in geometry. Electronic Journal of E-Learning, 17(1), pp52-63.

[3] Ahmadi, M. (2021). Effect of LEGO-Based Curriculum on Better Understanding of Mathematical Concepts among First-Grade Students. Turkish Journal of Computer and Mathematics Education (TURCOMAT), 12(13), 4801–4807.

[4] Anagnostopoulou, P., Alexandropoulou, V., Lorentzou, G., Lykothanasi, A., Ntaountaki, P., & Drigas, A. (2020). Artificial intelligence in autism assessment. International Journal of Emerging Technologies in Learning (IJET), 15(6), 95–107.

[5] Bamicha, V., & Drigas, A. (2022). The Evolutionary Course of Theory of Mind-Factors that facilitate or inhibit its operation & the role of ICTs. Technium Soc. Sci. J., 30, 138.

[6] Bashiru, A., & Nyarko, J. (2019). Van Hiele geometric thinking levels of junior high school students of Atebubu Municipality in Ghana. African Journal of Educational Studies in Mathematics and Sciences, 15(1), 39–50.

[7] Battista, M. T. (1990). Spatial visualization and gender differences in high school geometry. Journal for Research in Mathematics Education, 21(1), 47–60.

[8] Bhagat, K. K., & Chang, C.-Y. (2015). Incorporating GeoGebra into Geometry learning-A lesson from India. Eurasia Journal of Mathematics, Science and Technology Education, 11(1), 77–86.

[9] Biederman, I., & Cooper, E. E. (1991). Evidence for complete translational and reflectional invariance in visual object priming. Perception, 20(5), 585–593.

[10] Bindu, C. N. (2016). Impact of ICT on teaching and learning: A literature review. International Journal of Management and Commerce Innovations, 4(1), 24–31.

[11] Brainin, E., Shamir, A., & Eden, S. (2021). Robot programming intervention for promoting spatial relations, mental rotation and visual memory of kindergarten children. Journal of Research on Technology in Education, 1–14.

[12] Buckley, J., Seery, N., & Canty, D. (2018). A Heuristic Framework of Spatial Ability: a Review and Synthesis of Spatial Factor Literature to Support its Translation into STEM Education. In Educational Psychology Review (Vol. 30, Issue 3, pp. 947–972). Springer New York LLC. https://doi.org/10.1007/s10648-018-9432-z

[13] Burger, W. F., & Shaughnessy, J. M. (1986). Characterizing the van Hiele levels of development in geometry. Journal for Research in Mathematics Education, 17(1), 31–48.

[14] Carnoy, M. (2004). ICT in education: Possibilities and challenges. Inaugural Lecture of the UOC, 2005.

[15] Chaidi, I., & Drigas, A. (2022a). " Parents'' views Questionnaire for the education of emotions in Autism Spectrum Disorder" in a Greek context and the role of ICTs." Technium Soc. Sci. J., 33, 73.

[16] Chaidi, I., & Drigas, A. (2022b). Digital games & special education. Technium Soc. Sci. J., 34, 214.

[17] Chen, J., Li, L., & Zhang, D. (2021). Students With Specific Difficulties in Geometry: Exploring the TIMSS 2011 Data With Plausible Values and Latent Profile Analysis. Learning Disability Quarterly, 44(1), 11–22.

[18] Chen, L., Chen, P., & Lin, Z. (2020). Artificial intelligence in education: A review. Ieee Access, 8, 75264–75278.

[19] Chen, X., Xie, H., Zou, D., & Hwang, G.-J. (2020). Application and theory gaps during the rise of artificial intelligence in education. Computers and Education: Artificial Intelligence, 1, 100002.

[20] Chen, X., Zou, D., Xie, H., Cheng, G., & Liu, C. (2022). Two decades of artificial intelligence in education. Educational Technology & Society, 25(1), 28–47.

[21] Cheng, K., & Newcombe, N. S. (2005). Is there a geometric module for spatial orientation? Squaring theory and evidence. Psychonomic Bulletin & Review, 12, 1–23.

[22] Chiandetti, C., & Vallortigara, G. (2008). Is there an innate geometric module? Effects of experience with angular geometric cues on spatial re-orientation based on the shape of the environment. Animal Cognition, 11, 139–146.

[23] Das, K. (2019). The role and impact of ICT in improving the quality of education: An overview. International Journal of Innovative Studies in Sociology and Humanities, 4(6), 97–103.

[24] De Cruz, H. (2009a). An enhanced argument for innate elementary geometric knowledge and its philosophical implications. New Perspectives on Mathematical Practices. Essays in Philosophy and History of Mathematics, 185–206.

[25] De Cruz, H. (2009b). An enhanced argument for innate elementary geometric knowledge and its philosophical implications. New Perspectives on Mathematical Practices. Essays in Philosophy and History of Mathematics, 185–206.

[26] Dehaene, S., Izard, V., Pica, P., & Spelke, E. (2006). Core knowledge of geometry in an Amazonian indigene group. Science, 311(5759), 381–384. https://doi.org/10.1126/science.1121739

[27] Demertzi, E., Voukelatos, N., Papagerasimou, Y., & Drigas, A. S. (2018). Online learning facilities to support coding and robotics courses for youth. International Journal of Engineering Pedagogy (IJEP), 8(3), 69–80.

[28] Di Lieto, M. C., Castro, E., Pecini, C., Inguaggiato, E., Cecchi, F., Dario, P., Cioni, G., & Sgandurra, G. (2020). Improving executive functions at school in children with special needs by educational robotics. Frontiers in Psychology, 10, 2813.

[29] Dillon, M. R., Izard, V., & Spelke, E. S. (2020). Infants' sensitivity to shape changes in 2D visual forms. Infancy, 25(5), 618–639.

[30] Dorouka, P., Papadakis, S., & Kalogiannakis, M. (2020). Tablets and apps for promoting robotics, mathematics, STEM education and literacy in early childhood education. In Int. J. Mobile Learning and Organisation (Vol. 14, Issue 2).

[31] Doulou, A., & Drigas, A. (2022). Behavioral Problems in schools: A theoretical overview and the role of ICTs. Technium Soc. Sci. J., 29, 54.

[32] Drigas, A., & Charami, F. (2014). ICTs in English learning and teaching. International Journal of Recent Contributions from Engineering, Science & IT (IJES), 2(4), 4–10.

[33] Drigas, A., Dede, D. E., & Dedes, S. (2020). Mobile and other applications for mental imagery to improve learning disabilities and mental health. International Journal of Computer Science Issues (IJCSI), 17(4), 18–23.

[34] Drigas, A., & Dourou, A. (2013). A Review on ICTs, E-Learning and Artificial Intelligence for Dyslexicâ?? s Assistance. International Journal of Emerging Technologies in Learning (IJET), 8(4), 63–67.

[35] Drigas, A., Karyotaki, M., & Skianis, C. (2017). Success: A 9 layered-based model of giftedness. International Journal of Recent Contributions from Engineering, Science & IT (IJES), 5(4), 4–18.

[36] Drigas, A., & Kokkalia, G. (2014). ICTs in kindergarten. International Journal of Emerging Technologies in Learning (Online), 9(2), 52.

[37] Drigas, A., & Kostas, I. (2014). On Line and other ICTs Applications for teaching math in Special Education. International Journal of Recent Contributions from Engineering, Science & IT (IJES), 2(4), 46–53.

[38] Drigas, A., Mitsea, E., & Skianis, C. (2021). The Role of Clinical Hypnosis and VR in Special Education. International Journal of Recent Contributions from Engineering Science & IT (IJES), 9(4), 4–17.

[39] Drigas, A., Mitsea, E., & Skianis, C. (2022a). Intermittent Oxygen Fasting and Digital Technologies: from Antistress and Hormones Regulation to Wellbeing, Bliss and Higher Mental States. Technium BioChemMed, 3(2), 55–73.

[40] Drigas, A., Mitsea, E., & Skianis, C. (2022b). Subliminal Training Techniques for Cognitive, Emotional and Behavioral Balance. The Role of Emerging Technologies. Technium Soc. Sci. J., 33, 164.

[41] Drigas, A., Mitsea, E., & Skianis, C. (2022c). Virtual reality and metacognition training techniques for learning disabilities. Sustainability, 14(16), 10170.

[42] Drigas, A., & Papoutsi, C. (2021). Nine Layer Pyramid Model Questionnaire for Emotional Intelligence. International Journal of Online & Biomedical Engineering, 17(7).

[43] Drigas, A., Papoutsi, C., & Skianis, C. (2021). Metacognitive and Metaemotional Training Strategies through the Nine-layer Pyramid Model of Emotional Intelligence. International Journal of Recent Contributions from Engineering, Science & IT (IJES), 9(4), 58–76.

[44] Drigas, A., & Petrova, A. (2014). ICTs in speech and language therapy. International Journal of Engineering Pedagogy (IJEP), 4(1), 49–54.

[45] Drigas, A., & Rodi, E. I. (2013a). Special education and ICTs. International Journal of Emerging Technologies in Learning (Online), 8(2), 41.

[46] Drigas, A. S., & Ioannidou, R.-E. (2013). ICTs in special education: A review. Information Systems, E-Learning, and Knowledge Management Research: 4th World Summit on the Knowledge Society, WSKS 2011, Mykonos, Greece, September 21-23, 2011. Revised Selected Papers 4, 357–364.

[47] Drigas, A., & Sideraki, A. (2021). Emotional intelligence in autism. Technium Soc. Sci. J., 26, 80.

[48] Drigas, A., & Vlachou, J. A. (2016). Information and communication technologies (ICTs) and autistic spectrum disorders (ASD). International Journal of Recent Contributions from Engineering, Science & IT (IJES), 4(1), 4–10.

[49] Ferreirós, J., & García-Pérez, M. J. (2020a). Beyond natural geometry: on the nature of proto-geometry. Philosophical Psychology, 33(2), 181–205.

[50] Ferreirós, J., & García-Pérez, M. J. (2020b). Beyond natural geometry: on the nature of proto-geometry. Philosophical Psychology, 33(2), 181–205.

[51] Flecknoe, M. (2002). How can ICT help us to improve education? Innovations in Education and Teaching International, 39(4), 271–279.

[52] Galitskaya, V., & Drigas, A. (2019). ICTs and Geometry. Int. J. Eng. Pedagog., 9(5), 103–111.

[53] Galitskaya, V., & Drigas, A. (2020). Special education: Teaching geometry with ICTs. International Journal of Emerging Technologies in Learning (IJET), 15(6), 173–182.

[54] Galitskaya, V., & Drigas, A. (2021). The importance of working memory in children with Dyscalculia and Ageometria. Scientific Electronic Archives, 14(10).

[55] Giofrè, D., Mammarella, I. C., & Cornoldi, C. (2014). The relationship among geometry, working memory, and intelligence in children. Journal of Experimental Child Psychology, 123(1), 112–128. https://doi.org/10.1016/j.jecp.2014.01.002

[56] Giofré, D., Mammarella, I. C., & Cornoldi, C. (2014). The relationship among geometry, working memory, and intelligence in children. Journal of Experimental Child Psychology, 123, 112–128.

[57] Giofrè, D., Mammarella, I. C., Ronconi, L., & Cornoldi, C. (2013). Visuospatial working memory in intuitive geometry, and in academic achievement in geometry. Learning and Individual Differences, 23(1), 114–122. https://doi.org/10.1016/j.lindif.2012.09.012

[58] Gloria, C. C. (2015). Mathematical competence and performance in geometry of high school students. International Journal of Science and Technology, 5(2), 53–69.

[59] Gonsalves, N., & Krawec, J. (2014). Using number lines to solve math word problems: A strategy for students with learning disabilities. Learning Disabilities Research & Practice, 29(4), 160–170.

[60] Halat, E. (2008). Pre-Service Elementary School and Secondary Mathematics Teachers' Van Hiele Levels and Gender Differences. Issues in the Undergraduate Mathematics Preparation of School Teachers, 1.

[61] Halat, E., & Sahin, O. (2008). Van Hiele levels of pre-and in-service Turkish elementary school teachers and gender related differences in geometry. The Mathematics Educator, 11(1/2), 143–158.

[62] Haviger, J., & Vojkůvková, I. (2015). The Van Hiele levels at Czech secondary schools. Procedia-Social and Behavioral Sciences, 171, 912–918.

[63] Henderson, D. (2020). Benefits of ICT in Education. IDOSR Journal of Arts and Management. Pp. 1, 5.

[64] Hopcan, S., Polat, E., Ozturk, M. E., & Ozturk, L. (2023). Artificial intelligence in special education: a systematic review. Interactive Learning Environments, 31(10), 7335–7353.

[65] Huang, J., Saleh, S., & Liu, Y. (2021). A review on artificial intelligence in education. Academic Journal of Interdisciplinary Studies, 10(206).

[66] Hussain, S., Lindh, J., & Shukur, G. (2006). The effect of LEGO training on pupils' school performance in mathematics, problem solving ability and attitude: Swedish data. Journal of Educational Technology & Society, 9(3), 182–194.

[67] Hutchins, N., Darrah, T., Zare, H., & Biswas, G. (n.d.). A DSML for a robotics environment to support synergistic learning of CT and geometry.

[68] Izard, V., Sann, C., Spelke, E. S., & Streri, A. (2009). Newborn infants perceive abstract numbers. Proceedings of the National Academy of Sciences, 106(25), 10382–10385.

[69] Izard, V., & Spelke, E. S. (2009). Development of Sensitivity to Geometry in Visual Forms. Human Evolution, 23(3), 213–248.

[70] Julià, C., & Antolì, J. Ò. (2018). Enhancing spatial ability and mechanical reasoning through a STEM course. International Journal of Technology and Design Education, 28(4), 957–983.

[71] Kefalis, C., & Drigas, A. (2019). Web Based and Online Applications in STEM Education. International Journal of Engineering Pedagogy, 9(4).

[72] Kim, K. M., & Md-Ali, R. (2017). Geogebra: Towards realizing 21st century learning in mathematics education. Malaysian Journal of Learning and Instruction, 93–115.

[73] Kivkovich, N. (2015). A tool for solving geometric problems using mediated mathematical discourse (for teachers and pupils). Procedia-Social and Behavioral Sciences, 209, 519–525.

[74] Lean, G., & Clements, M. A. (1981). Spatial ability, visual imagery, and mathematical performance. Educational Studies in Mathematics, 12(3), 267–299.

[75] Lourenco, S. F., & Huttenlocher, J. (2008b). The representation of geometric cues in infancy. Infancy, 13(2), 103–127.

[76] Luan, H., Geczy, P., Lai, H., Gobert, J., Yang, S. J. H., Ogata, H., Baltes, J., Guerra, R., Li, P., & Tsai, C.-C. (2020). Challenges and future directions of big data and artificial intelligence in education. Frontiers in Psychology, 11, 580820.

[77] Lu'luilmaknun, U., Salsabila, N. H., Triutami, T. W., & Novitasari, D. (2021). The use of technology in learning geometry. Journal of Physics: Conference Series, 1778(1), 012030.

[78] Lytra, N., & Drigas, A. (2021). STEAM education-metacognition–Specific Learning Disabilities. Scientific Electronic Archives, 14(10).

[79] Majumdar, S. (2015). Emerging trends in ICT for education & training. IMPLEMENTATION OF ICT SKILLS FOR THE STUDENTS FOR THEIR EMPLOYMENT IN SKILL BASED JOBS, 1, 1.

[80] Mason, M. (2009). The van Hiele levels of geometric understanding. Colección Digital Eudoxus, 1(2).

[81] Mathematics, N. C. of T. of. (2009). Guiding principles for mathematics curriculum and assessment. News and Calendar: News.

[82] McDougal, E., Silverstein, P., Treleaven, O., Jerrom, L., Gilligan-Lee, K. A., Gilmore, C., & Farran, E. K. (2023a). Associations and indirect effects between LEGO® construction and mathematics performance. Child Development, 94(5), 1381–1397.

[83] McDougal, E., Silverstein, P., Treleaven, O., Jerrom, L., Gilligan-Lee, K., Gilmore, C., & Farran, E. K. (2023b). Assessing the impact of LEGO® construction training on spatial and mathematical skills. Developmental Science, e13432.

[84] Mensah, N. B., Barton Odro, E., & Williams, D. A. (2023). Examination of 9th Graders' Levels of Geometric Thinking. International Journal of Research in Education and Science, 9(3), 688–703.

[85] Mitsea, E., Drigas, A., & Skianis, C. (2022). ICTs and Speed Learning in Special Education: High-Consciousness Training Strategies for High-Capacity Learners through Metacognition Lens. Technium Soc. Sci. J., 27, 230.

[86] Mitsea, E., Lytra, N., Akrivopoulou, A., & Drigas, A. (2020). Metacognition, Mindfulness and Robots for Autism Inclusion. Int. J. Recent Contributions Eng. Sci. IT, 8(2), 4–20.

[87] Montague, M. (2003). Solve it! A mathematical problem solving instructional program. Reston, VA: Exceptional Innovations.

[88] Murti, S., & Szucs, D. (n.d.). How Is Building Lego Models Related to Math Skills?

[89] Nath, S., & Szücs, D. (2014a). Construction play and cognitive skills associated with the development of mathematical abilities in 7-year-old children. Learning and Instruction, 32, 73–80.

[90] Ng, D. T. K., Leung, J. K. L., Chu, K. W. S., & Qiao, M. S. (2021). AI literacy: Definition, teaching, evaluation and ethical issues. Proceedings of the Association for Information Science and Technology, 58(1), 504–509.

[91] Noor-Ul-Amin, S. (2013). An effective use of ICT for education and learning by drawing on worldwide knowledge, research, and experience. ICT as a Change Agent for Education. India: Department of Education, University of Kashmir, 1, 13.

[92] Nzaramyimana, E., Mukandayambaje, E., Iyamuremye, L., Hakizumuremyi, V., & Ukobizaba, F. (2021). Effectiveness of GeoGebra towards students' active learning, performance and interest to learn mathematics. International Journal of Mathematics and Computer Research, 9(10), 2423–2430.

[93] Ouyang, F., & Jiao, P. (2021). Artificial intelligence in education: The three paradigms. Computers and Education: Artificial Intelligence, 2, 100020.

[94] Pappas, M. A., Demertzi, E., Papagerasimou, Y., Koukianakis, L., Kouremenos, D., Loukidis, I., & Drigas, A. S. (2018). E-learning for deaf adults from a user-centered perspective. Education Sciences, 8(4), 206.

[95] Pappas, M. A., Demertzi, E., Papagerasimou, Y., Koukianakis, L., Voukelatos, N., & Drigas, A. (2019). Cognitive-based E-learning design for older adults. Social Sciences, 8(1), 6.

[96] Pearce, D. L., Bruun, F., Skinner, K., & Lopez-Mohler, C. (2013). What teachers say about student difficulties solving mathematical word problems in grades 2-5. International Electronic Journal of Mathematics Education, 8(1), 3–19.

[97] Peng, Y., Deng, B., Zhang, J., Geng, F., Qin, W., & Liu, L. (2018). Anderson acceleration for geometry optimization and physics simulation. ACM Transactions on Graphics (TOG), 37(4), 1–14.

[98] Pica, P., Lemer, C., Izard, V., & Dehaene, S. (2004). Exact and approximate arithmetic in an Amazonian indigene group. Science, 306(5695), 499–503.

[99] Pongsakdi, N., Kajamies, A., Veermans, K., Lertola, K., Vauras, M., & Lehtinen, E. (2020). What makes mathematical word problem solving challenging? Exploring the roles of word problem characteristics, text comprehension, and arithmetic skills. ZDM, 52, 33–44.

[100] Pratama, M. P., Sampelolo, R., & Lura, H. (2023). Revolutionizing education: harnessing the power of artificial intelligence for personalized learning. Klasikal: Journal of Education, Language Teaching and Science, 5(2), 350–357.

[101] Prayito, M., Suryadi, D., & Mulyana, E. (2019). Geometric thinking level of the Indonesian seventh grade students of junior high school. Journal of Physics: Conference Series, 1188(1), 012036.

[102] Rachmaniah Mirza Hariastuti, M. (2023). Van Hiele's Theory: Transforming And Gender Perspective of Student's Geometrical Thinking. Journal of Physics: Conference Series, 2020, 1–2.

[103] Rashevska, N., Semerikov, S., Zinonos, N., Tkachuk, V., & Shyshkina, M. (2020). Using augmented reality tools in the teaching of two-dimensional plane geometry.

[104] Rejeki, S., Setyaningsih, N., & Toyib, M. (1848). Using LEGO for Learning Fractions, Supporting or Distracting? AIP Conference Proceedings, 40016(2017).

[105] Rosch, E. (1975). Cognitive representations of semantic categories. Journal of Experimental Psychology: General, 104(3), 192.

[106] Saads, S., & Davis, G. (1997). Spatial abilities, van Hiele levels & language use in three dimensional geometry. PME CONFERENCE, 4, 4–104.

[107] Sisman, B., Kucuk, S., & Yaman, Y. (2021). The effects of robotics training on children's spatial ability and attitude toward STEM. International Journal of Social Robotics, 13(2), 379–389.

[108] Škrbec, M., & Čadež, T. H. (2015). Identifying and fostering higher levels of geometric thinking. EURASIA Journal of Mathematics, Science and Technology Education, 11(3), 601–617.

[109] Slater, A., Mattock, A., Brown, E., & Bremner, J. G. (1991). Form perception at birth: Revisited. Journal of Experimental Child Psychology, 51(3), 395–406.

[110] Slater, A., Morison, V., Somers, M., Mattock, A., Brown, E., & Taylor, D. (1990). Newborn and older infants' perception of partly occluded objects. Infant Behavior and Development, 13(1), 33–49.

[111] Spelke, E., Lee, S. A., & Izard, V. (2010). Beyond core knowledge: Natural geometry. Cognitive Science, 34(5), 863–884. https://doi.org/10.1111/j.1551-6709.2010.01110.x

[112] Spelke, E. S. (2011). Natural number and natural geometry. In Space, time and number in the brain (pp. 287–317). Elsevier.

[113] Spencer, J. P., & Hund, A. M. (2003). Developmental continuity in the processes that underlie spatial recall. Cognitive Psychology, 47(4), 432–480.

[114] Stathopoulou, A., Karabatzaki, Z., Tsiros, D., Katsantoni, S., & Drigas, A. (2019). Mobile apps the educational solution for autistic students in secondary education.

[115] Su, J., & Yang, W. (2022). Artificial intelligence in early childhood education: A scoping review. Computers and Education: Artificial Intelligence, 3, 100049.

[116] Sumarwati, S., Fitriyani, H., Setiaji, F. M. A., Amiruddin, M. H., & Jalil, S. A. (2020). Developing Mathematics Learning Media Based on E-Learning using Moodle on Geometry Subject to Improve Students' Higher Order Thinking Skills. Int. J. Interact. Mob. Technol., 14(4), 182–191.

[117] Swanson, H. L. (2015). Cognitive strategy interventions improve word problem solving and working memory in children with math disabilities. Frontiers in Psychology, 6, 1099.

[118] Swanson, H. L. (2016). Word problem solving, working memory and serious math difficulties: Do cognitive strategies really make a difference? Journal of Applied Research in Memory and Cognition, 5(4), 368–383.

[119] Swanson, H. L., Lussier, C. M., & Orosco, M. J. (2015). Cognitive strategies, working memory, and growth in word problem solving in children with math difficulties. Journal of Learning Disabilities, 48(4), 339–358.

[120] Tsuruhara, A., Kanazawa, S., & Yamaguchi, M. K. (2009). Infants can't discriminate the orientation of a grating surrounded by an oblique square. Perception, 38(7), 1035–1044.

[121] Usiskin, Z. (1982). Van Hiele levels and achievement in secondary school geometry.

[122] Vallortigara, G., Sovrano, V. A., & Chiandetti, C. (2009). Doing Socrates experiment right: controlled rearing studies of geometrical knowledge in animals. Current Opinion in Neurobiology, 19(1), 20–26.

[123] Verstijnen, I. M., van Leeuwen, C., Goldschmidt, G., Hamel, R., & Hennessey, J. M. (1998). Creative discovery in imagery and perception: Combining is relatively easy, restructuring takes a sketch. Acta Psychologica, 99(2), 177–200.

[124] Vincent-Lancrin, S., & Van der Vlies, R. (2020). Trustworthy artificial intelligence (AI) in education: Promises and challenges.

[125] Wolfgang, C. H., Stannard, L. L., & Jones, I. (2001). Block play performance among preschoolers as a predictor of later school achievement in mathematics. Journal of Research in Childhood Education, 15(2), 173–180.

[126] Wolfgang, C., Stannard, L., & Jones, I. (2003). Advanced constructional play with LEGOs among preschoolers as a predictor of later school achievement in mathematics. Early Child Development and Care, 173(5), 467–475.

[127] Xanthopoulou, M., Kokalia, G., & Drigas, A. (2019). Applications for Children with Autism in Preschool and Primary Education. Int. J. Recent Contributions Eng. Sci. IT, 7(2), 4–16.

[128] Xie, J. (2020). On the exploration of a mobile executive functioning coaching solution for students with and without disabilities in post-secondary STEM education. International Journal of Mobile Learning and Organisation, 14(2), 136–160.

[129] Yang, W. (2022). Artificial Intelligence education for young children: Why, what, and how in curriculum design and implementation. Computers and Education: Artificial Intelligence, 3, 100061.

[130] Zaranis, N. (2014). Geometry teaching through ICT in primary school. In Research on e-Learning and ICT in Education: Technological, Pedagogical and Instructional Perspectives (pp. 59–71). Springer.

[131] Zhai, X., Chu, X., Chai, C. S., Jong, M. S. Y., Istenic, A., Spector, M., Liu, J.-B., Yuan, J., & Li, Y. (2021). A Review of Artificial Intelligence (AI) in Education from 2010 to 2020. Complexity, 2021, 1–18.

[132] Zhang, D., Ding, Y., Stegall, J., & Mo, L. (2012). The effect of visual-chunking-representation accommodation on geometry testing for students with math disabilities. Learning Disabilities Research & Practice, 27(4), 167–177.

[133] Zhong, B., & Xia, L. (2020). A Systematic Review on Exploring the Potential of Educational Robotics in Mathematics Education. International Journal of Science and Mathematics Education, 18(1), 79–101. https://doi.org/10.1007/s10763-018-09939-y

[134] Zweekhorst, M. B. M., & Maas, J. (2015). ICT in higher education: Students perceive increased engagement. Journal of Applied Research in Higher Education, 7(1), 2–18